Key

Graphing Rational Functions Worksheet

For the following function, find:

(a) the x-intercepts, y-intercepts
(b) the vertical asymptote(s)
(c) the horizontal asymptote
(d) the holes
(e) any additional points needed
(f) then, graph the function.

1. $f(x)=\dfrac{2x}{x^2-1}$

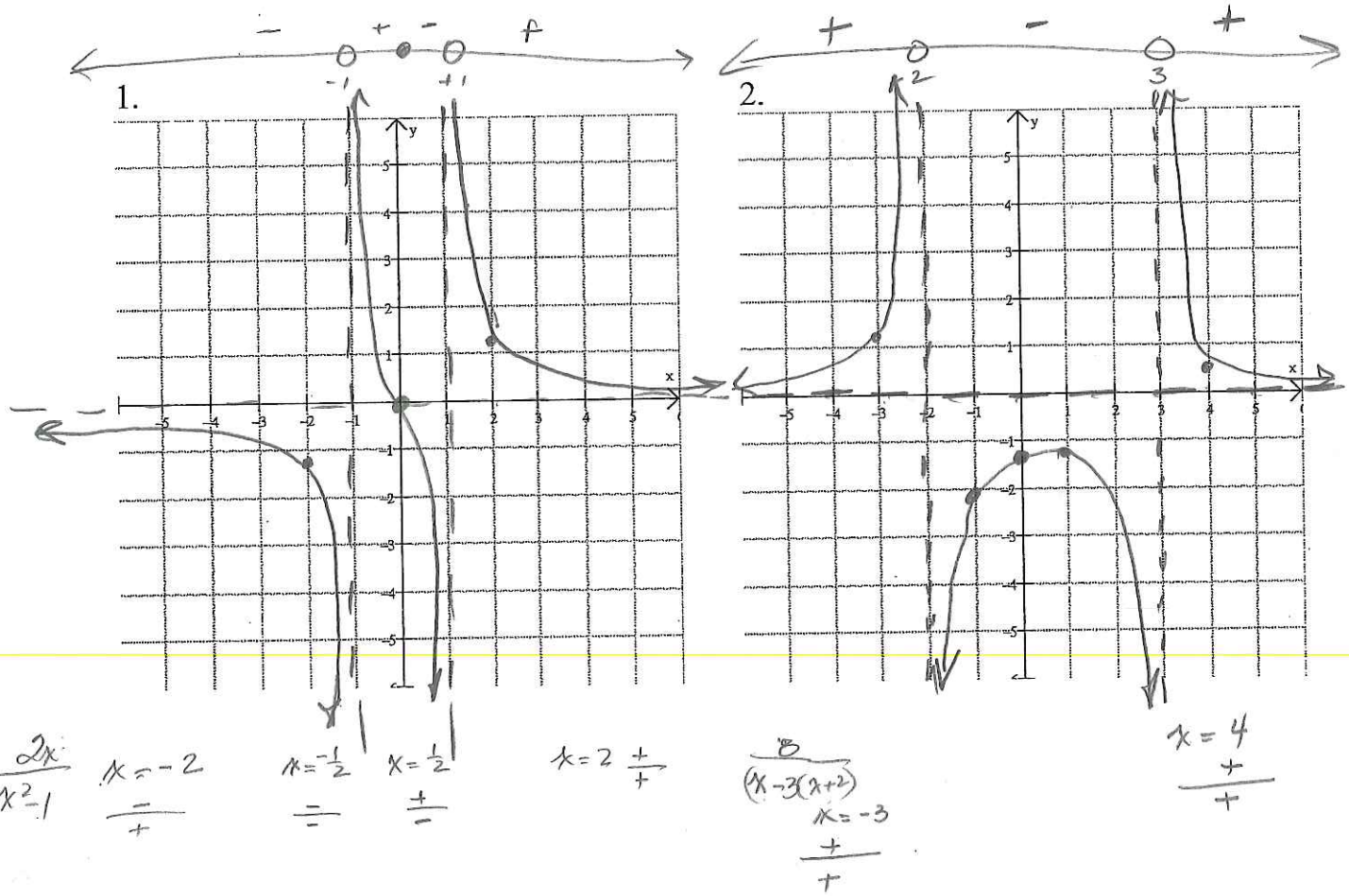
2. $y=\dfrac{8}{x^2-x-6}$

3. $f(x)=\dfrac{x^2-9}{2x^2+1}$

4. $y=\dfrac{x^2-5x+6}{x^2-4x+3}$

5. $y=\dfrac{x^2+11x+18}{2x+1}$

6. $g(x)=\dfrac{x-4}{x^2-3x}$



1. $f(x) = \frac{2x}{(x^2-1)}$

Domain x - any real # $x \neq -1, x \neq +1$

VA $x = -1, x = +1$

No holes

x-int $2x = 0$, $x = 0$

$(0,0)$ zero

y-int $(0,0)$

HA $y = 0$

Critical points $x = 0, x = -1, x = +1$

Check pts for graph

$x = -2 \quad \frac{(-4)}{4-1} = -\frac{4}{3}$

$x = 2 \quad \frac{4}{4-1} = \frac{4}{3}$

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2) $y = \frac{8}{x^2-x-6} = \frac{8}{(x-3)(x+2)}$

Domain $x \in \mathbb{R}, x \neq 3, x \neq -2$

VA $x = 3, x = -2$

No holes

x-int - none No zeros

y-int $y = -\frac{4}{3}$ $(0, -\frac{4}{3})$

H.A. $y = 0$

Critical points $x = 3, x = -2$

Check pts

$x = -3 \quad y = \frac{8}{9+3-6} = \frac{8}{6} = \frac{4}{3}$

$x = -1 \quad y = \frac{8}{1+1-6} = -2$

$x = 1 \quad y = \frac{8}{1-1-6} = -\frac{4}{3}$

$x = 5 \quad y = \frac{8}{25-5-6} = \frac{8}{14} = \frac{4}{7}$

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3) $f(x) = \frac{x^2-9}{2x^2+1}$

Domain x - any real #

VA - none

Holes - none

x int zeros $x^2-9=0$, $x = 3, x = -3$

$(-3,0)(3,0)$

y int $(0,-9)$

HA $y = \frac{1}{2}$

Critical points $x = 3, x = -3$

Check points

$x = -4 \quad y = \frac{16-9}{35} = \frac{7}{35} = \frac{1}{5}$

$x = -6 \quad y = \frac{36-9}{}$

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4) $y = \frac{(x^2-5x+6)}{(x^2-4x+3)} = \frac{(x-3)(x-2)}{(x-3)(x-1)}$

Domain $x \in \mathbb{R}, x \neq 3, x \neq 1$

VA $x = 1$

Hole at $x = 3$ $(3, \frac{1}{2})$

x-int zero @ $x = 2$

$(2,0)$

y int $(0,2)$

H.A $y = 1$

Critical points

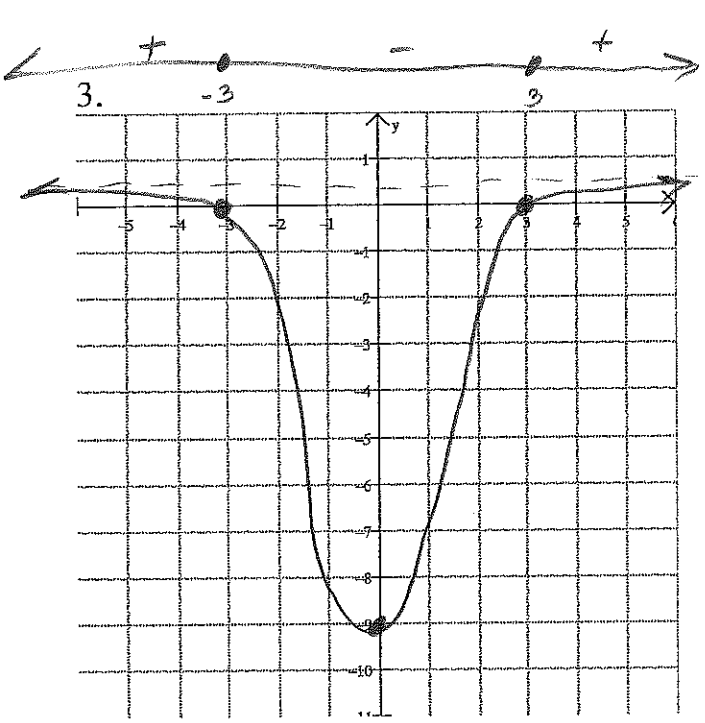
$x = 3, x = 1, x = 2$

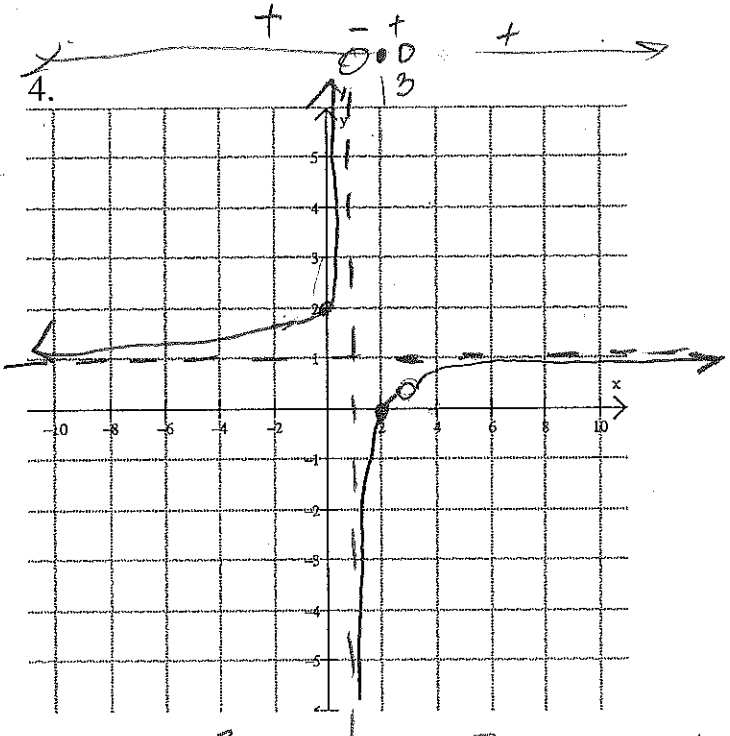
Check pts

$x = -5 \quad y = \frac{-7}{-6}$

$x = -3 \quad y = \frac{-5}{-4}$

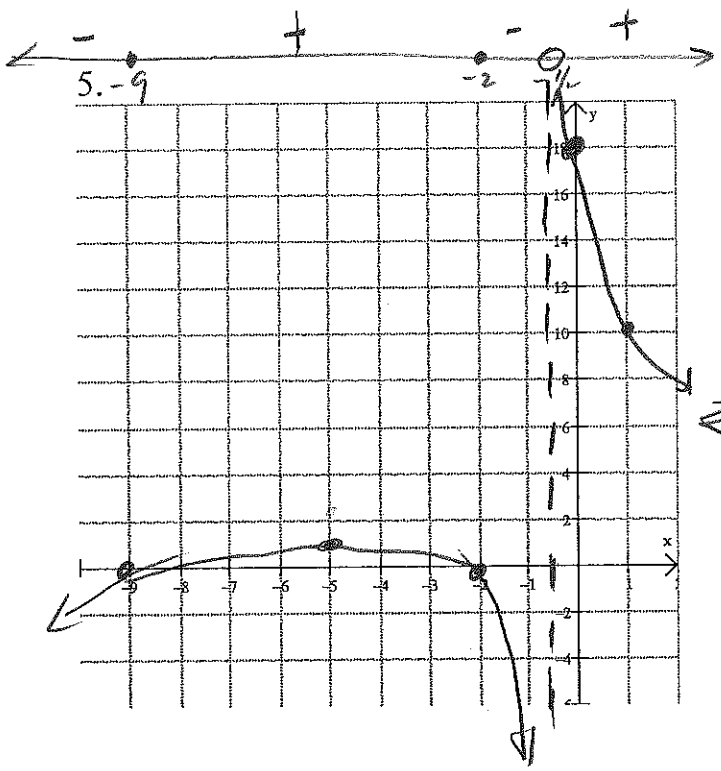
$x = 5 \quad y = \frac{3}{4}$

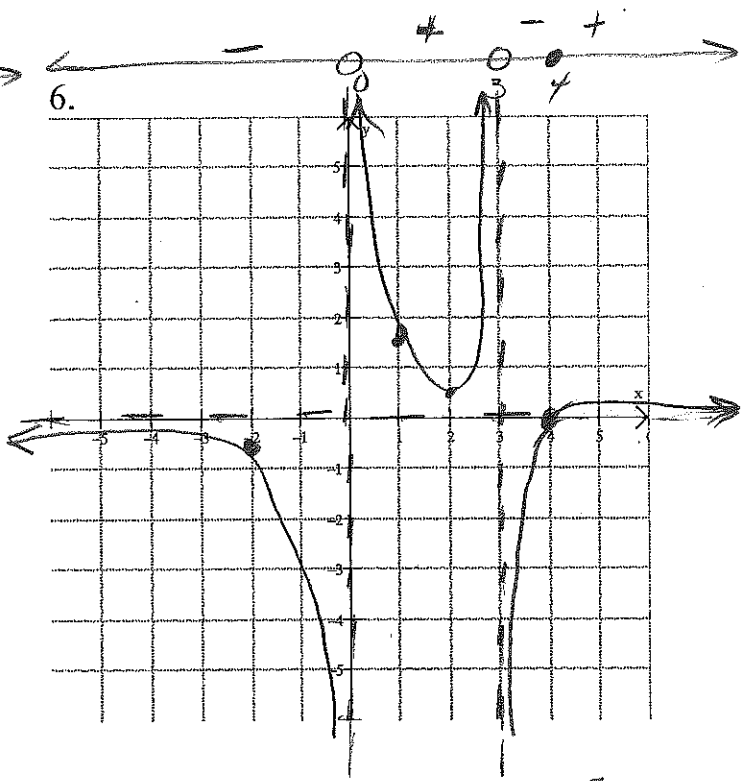




$\frac{x-2}{x-1}$ $x=\frac{3}{2}$ $\frac{-}{+}$ $x=\frac{5}{2}$ $\frac{+}{+}$ $x=1$ $\frac{+}{+}$

a





$\frac{x-4}{x(x-3)}$ $x=-2$ $\frac{-}{-(-)}$ $x=1$ $\frac{-}{+(-)}$ $x=\frac{7}{2}$ $\frac{-}{+(+)}$

$x=5$ $\frac{+}{+}$

3) $y = \frac{x^2+11x+18}{2x+1} = \frac{(x+9)(x+2)}{2x+1}$

Domain $x \in \mathbb{R}$ $x \neq -\frac{1}{2}$

VA $x = -\frac{1}{2}$

NO HOLES

x-int zeros @ $x = -9$, $x = -2$

$(-9, 0)$ $(-2, 0)$

y-int

$(0, 18)$

H.A. none (slant asymptote)

critical points $x = \frac{1}{2}$ $x = -9$, $x = -2$

Check pts

$x = -5$ $y = \frac{(4)(-3)}{-9} = \frac{12}{9} = \frac{4}{3}$

$x = 1$ $y = \frac{(10)(3)}{3} = 10$

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6) $g(x) = \frac{x-4}{x^2-3x} = \frac{x-4}{x(x-3)}$

D: $x \in \mathbb{R}$, $x \neq 0$, $x \neq 3$

VA $x = 0$, $x = 3$

NO HOLES

x-int zeros @ $x = 4$

$(4, 0)$

y-int NONE

H.A. $y = 0$

critical points $x = 0$, $x = 3$, $x = 4$

Check pts

$x = -2$ $y = \frac{-6}{-2(-5)} = \frac{-6}{10} = -\frac{3}{5}$

$x = 1$ $y = \frac{-3}{1(-2)} = \frac{3}{2}$

$x = 2$ $y = \frac{-2}{2(-2)} = \frac{1}{2}$

$x = 5$ $y = \frac{1}{5(2)} = \frac{1}{10}$